

# Age-structured compartmental epidemic models for Northern Ireland

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## Introduction

Age-structured SEIIR models with five 20-year age classes can simulate COVID-19 in the Northern Ireland population during 2020-21. They may then be used to determine optimal lockdown strategies that will minimize total deaths or peak hospital occupancy, using hospital occupancy as a trigger.

## Model description

The population is divided into eight compartments for each of the five age classes  $i$ .

Infections are controlled by time-dependent basic transmission rate  $\beta$  and the age contact matrix  $c_{i,j}$ . They are initially subclinical, before developing symptoms with age-dependent probability  $\epsilon_i$ .

Rates of change for the compartments:

- Susceptible:  $\frac{dS_i}{dt} = -\beta \alpha_i S_i P_i^{-1} \sum_{j=1}^5 c_{j,i} (i_S I_{S,j} + i_C I_{C,j})$
- Exposed:  $\frac{dE_i}{dt} = \beta \alpha_i S_i P_i^{-1} \sum_{j=1}^5 c_{j,i} (i_S I_{S,j} + i_C I_{C,j}) - \sigma E_i$
- Infectious (subclin.):  $\frac{dI_{S,i}}{dt} = \sigma E_i - \gamma_S I_{S,i}$
- Infectious (clinical):  $\frac{dI_{C,i}}{dt} = \epsilon_i \gamma_S I_{S,i} - \gamma_C I_{C,i}$
- Hospitalised:  $\frac{dH_{1,i}}{dt} = h_{1,i} \gamma_C I_{C,i} - \delta_1 H_{1,i}$
- ICU:  $\frac{dH_{2,i}}{dt} = h_{2,i} \delta_1 H_{1,i} - \delta_2 H_{2,i}$
- Deceased:  $\frac{dD_i}{dt} = d_{1,i} \delta_1 H_{1,i} + d_{2,i} \delta_2 H_{2,i}$
- Recovered:  $\frac{dR_i}{dt} = (1 - \epsilon_i) \gamma_S I_{S,i} + (1 - h_{1,i}) \gamma_C I_{C,i} + (1 - h_{2,i} - d_{1,i}) \delta_1 H_{1,i} + (1 - d_{2,i}) \delta_2 H_{2,i}$

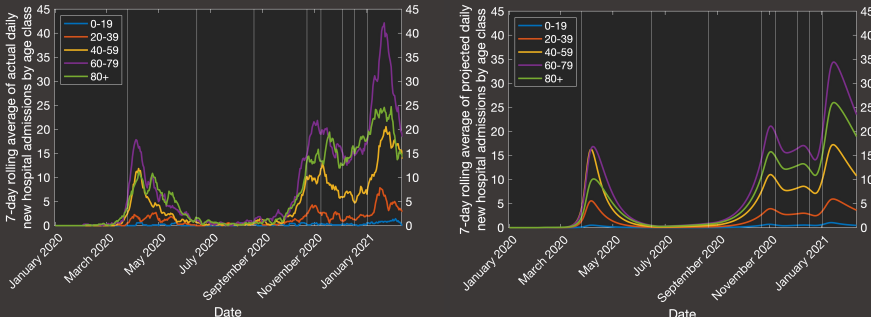
Figure 1: Model flowchart (within age-classes).

These are solved numerically using the Euler method.

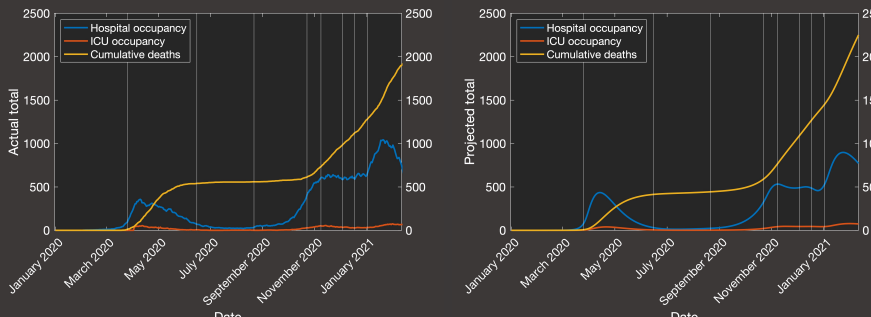
Link to parameter sources:



## Fitted simulation

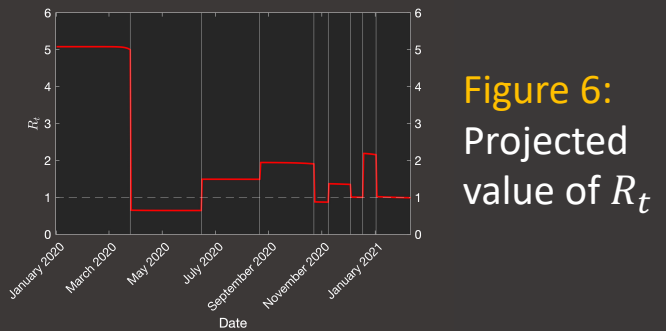


Figures 2-3: Empirical and fitted daily hospital admissions (7-day rolling average).



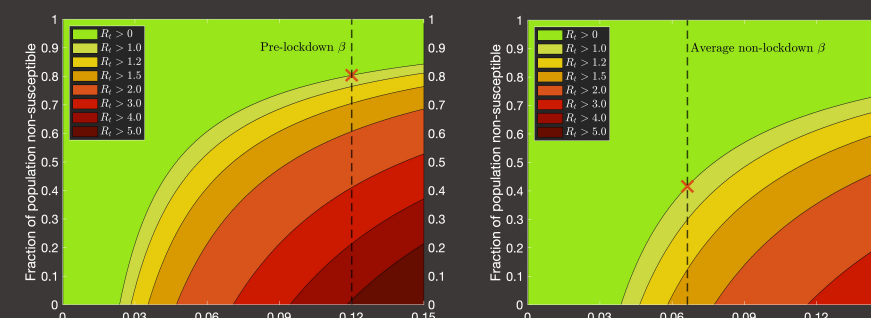
Figures 4-5: Empirical and projected deaths, ICU and hospital occupancies.

Transmission rates  $\beta$  during nine time-periods from January 2020 to February 2021 are selected by fitting daily hospital admissions (by age class) to [empirical data](#) (QR-code) from the Department of Health.



- Results:
- 9.5% of the NI population had contracted COVID-19 by Feb. 2021.
  - $R_0 \approx 5$
  - Winter 2020 circuit breaks were less effective than the initial lockdown in reducing  $R_t$ .

## Immunity and $R_t$

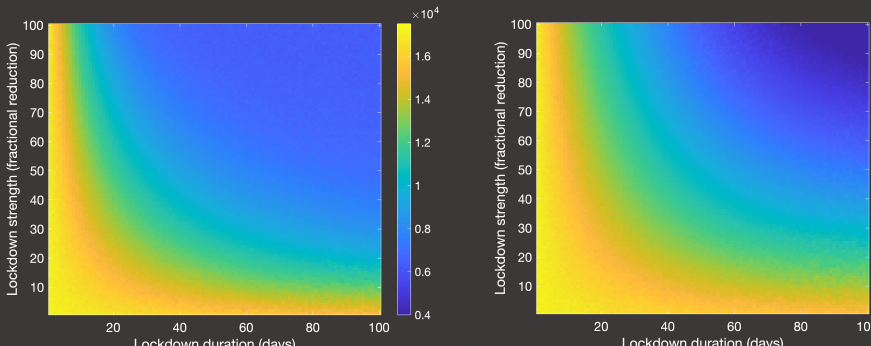


Figures 7-8:  $R_t$  thresholds pre-pandemic (early 2020) and post-lockdown (mid-2021).

- Calculating the effective reproduction ratio  $R_t$  as a function of  $\beta$  and the immune fraction of the population.
- In March 2020, we would require 80.4% of the population to be immune to achieve  $R_t < 1$ .
  - In 2020/21 average non-lockdown conditions (with reduced social contact), this is lowered to 42%.

## Lockdown intensity and duration

How does the cumulative number of deaths depend on the intensity (y-axis) and duration (x-axis) of a single optimal lockdown within a 1000-day simulation, assuming no vaccination programme? (3476 COVID-related deaths were recorded by October 2021.)



Figures 9-10: Total deaths given optimally-timed lockdowns of the whole population or 60+ only.

- Results:
- Deaths can be reduced from 16000 to 4000 in a closed system (or 7000 if there is constant inflow of infections).
  - Interventions targeting the vulnerable are optimal, but *only* if sufficiently strong.

## Mechanistic lockdown triggers

If lockdowns are triggered by breaching a given threshold of hospital occupancy (between 0-2000), what is the optimal threshold and what is the effect of a delay (up to 20 days) before the intervention is implemented?

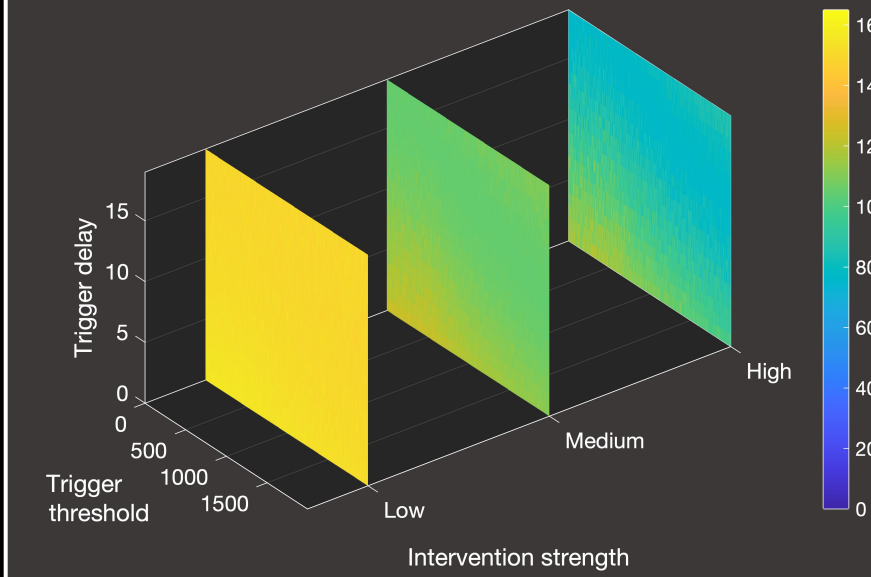


Figure 11: Total deaths in a 2000-day simulation with one dynamically-triggered lockdown of strength: Low: 25% for 15 days, Medium: 50% for 30 days, High: 75% for 60 days. A single lockdown can significantly reduce deaths on this timescale even with constant new infections.

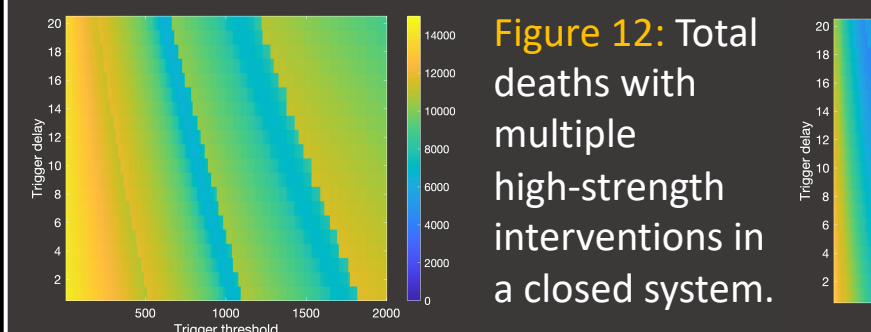


Figure 12: Total deaths with multiple high-strength interventions in a closed system.

Earlier lockdown timings/shorter delays are preferable to reduce peak hospital occupancy, while slightly later lockdowns better reduce total deaths. This holds when permitting either one or multiple interventions during the first 1000 simulation days.

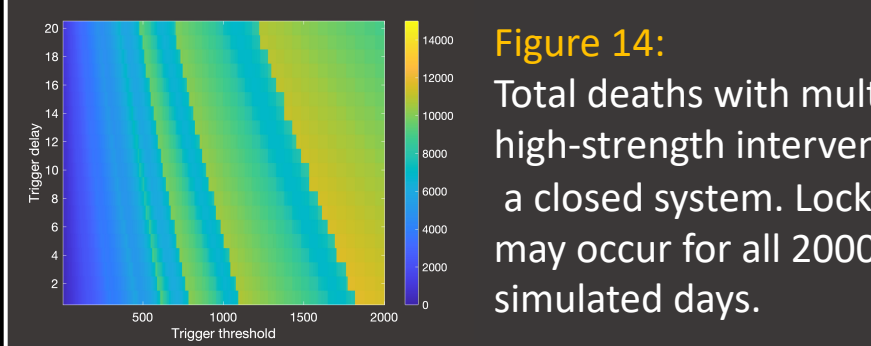


Figure 13: Peak hospital occupancy with multiple high-strength interventions in a closed system.

## Conclusion

A single lockdown, properly timed using the current number of hospital occupants as a trigger mechanism, can significantly reduce the total number of deaths for a long period of time - even assuming that the virus is not fully eliminated. Optimal use can minimize long-term deaths while a vaccination exit strategy is developed.

- Restrictions that only target the most vulnerable may be as or more effective, but only provided that they are intense and endure long enough to enable immunity amongst the rest of the population.
- Implementing the strongest controls too early in a pandemic may have negative consequences when they are finally released, with a potentially greater number of deaths in the long term than if it was allowed to spread first.