Dynamics and stability in meta-population models

Dr Gavin M Abernethy

Computing Science and Mathematics,
University of Stirling

2023

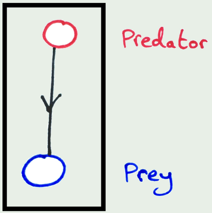
Population models: introduction

We may be familiar with two-species models in population dynamics:

Predator-prey model

Predator: $$\frac{\mathrm{d}y}{\mathrm{d}t} = \lambda g(x,y)y - dy$$

Prey: $$\frac{\mathrm{d}x}{\mathrm{d}t} = f(x)x - g(x,y)y$$



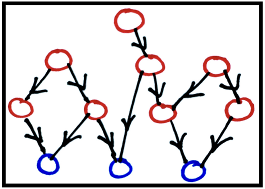
e.g. with logistic prey growth and Lotka-Volterra predation...

Predator: $$\frac{\mathrm{d}y}{\mathrm{d}t} = \lambda cxy - dy$$

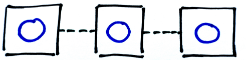
Prey: $$\frac{\mathrm{d}x}{\mathrm{d}t} = rx(1-x) - cxy$$

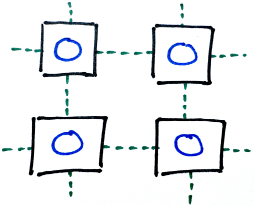
Population models: extensions

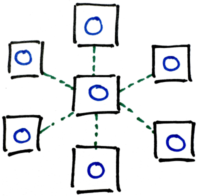
- **Community model:** multiple interacting species.



- **Meta-population model:** single species, multiple patches.







Applications overview

Modelling framework:

Systems of coupled ODE's, arranged on a discretised space-, age-, or group-structured network.

Applications:

I. **Ecological networks:** competing and predating species on a spatial network...
   - (i) Abstract species with evolutionary dynamics;
   - (ii) Applied models of particular species and places;

II. **Epidemiology:** COVID-19 in the age-structured population of Northern Ireland.

III. **Opinion dynamics:** between and amongst social groups.

Simulating the evolutionary assembly and spread of an ecological **meta-community** in space from first species.

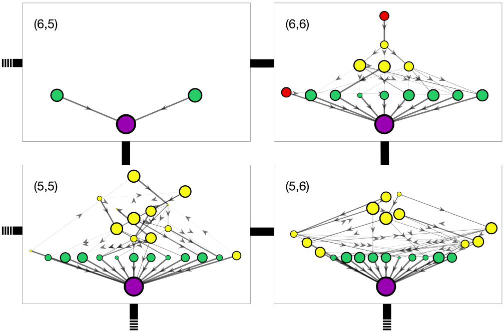


Figure 1: 4 of 6×6 patches in an ecological meta-community

For each local population N:

$$\frac{\mathrm{d}N}{\mathrm{d}t} = F - M - P + \mu_i - \mu_e$$

where

- F = gains due to feeding;
- M = natural mortality;
- P = loss due to predation;
- μ_e, μ_i = dispersal to/from neighbours;

Mutants of existing species invade periodically.

Application I(i): Species extinction and invasion

Applied to questions of stability and resilience to climate change, species extinctions and invasions:

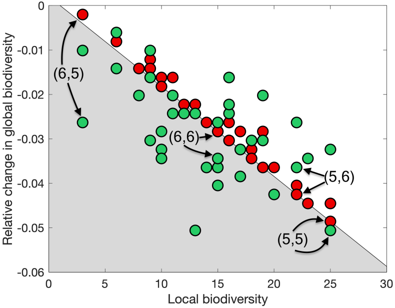


Figure 2: Effect: Displacement vs. elimination

From a meta-community of 458 species in 6×6 patches, *displacement* to neighbouring patches *can be more damaging than their extinction*.

In a similar experiment (7000 species in 15 M-C's):

- Each species is deleted, (max loss 2.1%).
- then re-introduced to *all* patches (max loss 12%).

Application I(ii): Water vole and mink

Management of water vole and mink populations (with Sheffield Hallam University and Sheffield & Rotherham Wildlife Trust):

- Two-species meta-population dynamics;
- Spatial network constructed from existing GIS mappings;
- Predicting impact of:
  - removing patches
  - altering habitat types
  - wildlife corridors connecting patches
- Where to place reserves?

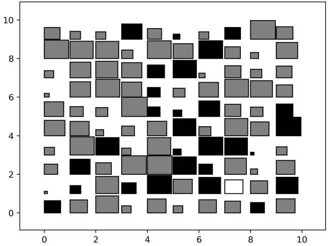


Figure 3: Biodiversity in a 10×10 test environment

Application II: Epidemic models for Northern Ireland

Age-structured epidemic models split a population into discrete age-classes - functionally similar to discrete spatial sub-populations.

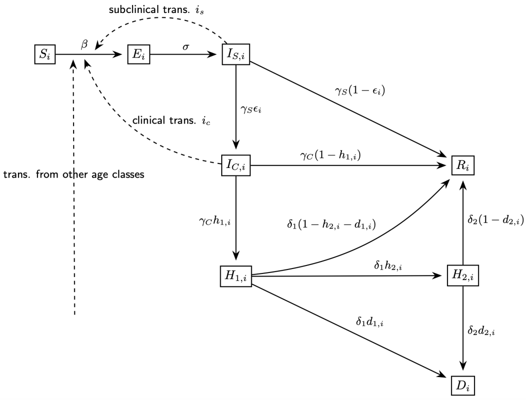
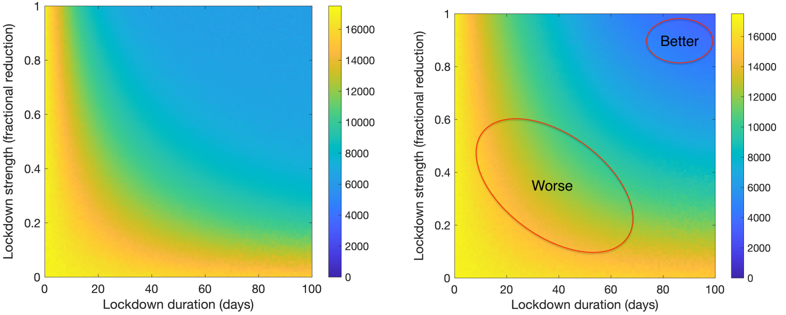


Figure 4: SEIIR model for one of the age-classes

Calibrating such an SEIIR model for COVID-19 in NI, we study optimal lockdown timing and duration.

Result: without a vaccine exit strategy, locking down too strong and early in an isolated community leads to later resurgence with worse overall outcomes.

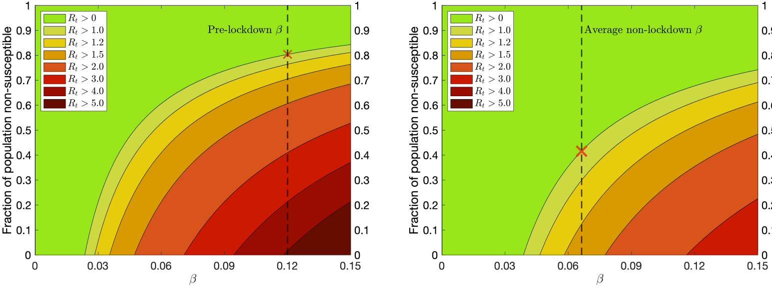
Application II: Optimal lockdowns



(a) Whole population (b) Over 60's only

Figure 5: Cumulative deaths with optimally-timed single lockdown

Single lockdowns targeting the more vulnerable may be more effective, but *only* if sufficient that the virus spreads then dissipates in the remaining population (possible due to age-structured model).



(a) Pre-pandemic 2020 conditions (b) Early 2021 non-lockdown conditions

Figure 6: Dependence of R_t on immune fraction of the population

From average transmission rates in Spring 2020 (Fig. 6(a)) and Autumn 2020 - Spring 2021 outside of lockdowns (Fig. 6(b)):

80.4% or 42% must be immune to prevent spread ($R_t < 1$).

Application III: Opinion dynamics in social groups

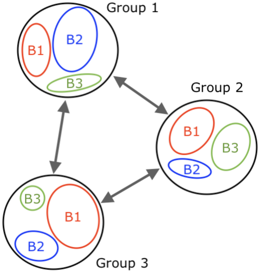


Figure 7: Multiple beliefs in coupled social groups

Spread of competing beliefs/affiliations in a network of social groups:

- Media influencers;
- Misinformation spread;
- Competing ideologies - consensus or political divergence?

For a smaller-scale, individual-based Hegselmann-Krause models can be parameterised based on real social network data.
Then we may modify the network structure to create new connections or sever existing ones[1]. How can "true news" be targeted?

[1] Equivalent to spatial network perturbations and wildlife corridors.

References

- Abernethy, G. M. (2022). Perturbation responses in co-evolved model meta-communities. *Ecology and Evolution*, 12.11: e9534.
- Abernethy, G.M. and Glass D.H. (2022). Optimal COVID-19 lockdown strategies in an age-structured SEIR model of Northern Ireland. *Journal of the Royal Society Interface*, 19:188
- McCartney, M. and Glass D.H. (2015). The dynamics of coupled logistic social groups. *Physica A*, 427:141-154.
- **Review:** Gross et al. (2020). Modern models of trophic meta-communities. *Philosophical transactions of the Royal Society B*, 375: 20190455.