FMSS: Lecture 2 handout

Delayed step functions

$U(t-???)$ is a step function with delay.

Function $f(t)$ takes a value of E only between times $t = T_1$ and $t = T_2$:

$$f(t) = E\Big(U(t - T_1) - U(t - T_2)\Big)$$

Delay theorem

To take the Laplace transform of a function with time-delay of value T:

$$\mathcal{L}\Big\{g(t - T)U(t - T)\Big\} = e^{-sT}\mathcal{L}\Big\{g(t)\Big\}$$

Writing in delay form

A delayed function is in delay form if all occurences of t are written explicitly as $t - T$, where T is the delay. If not, we can achieve this by replacing all occurrences of t with $(t - T) + T$.

Applying the delay theorem to transform a delayed function

To take the Laplace transform of a function of the form:

$$f(t) = g(t - T)U(t - T)$$

1. Write the function in delay form.
2. Name the part multiplied by the step function as $g(t - delay)$.
3. Replace $t - delay$ with t to obtain the function $g(t)$.
4. Take the Laplace transform of $g(t)$.
5. Multiply by $\mathrm{e}^{-s\times\mathrm{delay}}$

Inverse delay theorem

To take the inverse Laplace transform of a function with time-delay of value T:

$$\mathcal{L}^{-1}\left\{e^{-sT}\bar{g}(s)\right\} = g(t-T)U(t-T)$$

Applying the inverse delay theorem

To take the inverse Laplace transform of a function of the form:

$$\bar{f}(s) = \bar{g}(s)e^{-sT}$$

1. Identify that the factor e^{-sT} means there will be a delay of T.
2. Identify that the other part is $\bar{g}(s)$, where $\bar{f}(s) = \bar{g}(s)e^{-sT}$.
3. Invert this part to obtain $g(t) = \mathcal{L}^{-1}\{\bar{g}(s)\}$.
4. Change the variable from t to $t-T$.

   Thus replace every occurrence of t in $g(t)$ with $t-T$, to get the function $g(t-T)$.
5. Multiply by the step function $U(t-T)$, to get:

$$f(t) = g(t-T)U(t-T)$$