FMSS: Lecture 4 handout

Transfer function of a system

Given a linear system with input $v_i(t)$ and output $v_o(t)$, the transfer function is:

$$G(s) = \frac{\bar{v}_o(s)}{\bar{v}_i(s)}$$

Determining the transfer function

Given a set of multiple ODEs corresponding to the system, to find the transfer function:

1. Take Laplace transforms of all equations.
2. Plan a sequence of steps (substituting and rearranging the equations) to eliminate all other s-dependent variables.
3. Enact the plan!
4. Re-arrange to the form: $\bar{v}_o(s) = G(s) \times \bar{v}_i(s)$.
5. Read off $G(s)$.

Characteristic equation

With the transfer function $G(s)$ simplified as much as possible, the characteristic equation is given by:

$$\text{denominator} \quad \text{of} \quad G(s) \quad = \quad 0$$

Order of the system

The largest power of s (equivalently, the number of solutions) in the characteristic equation is the order of the system. (e.g. if it is a quadratic equation, then it is "second-order").

Stability of the system

A system is stable **if and only if** **all** solutions of the characteristic equation have **negative real part**.

Example 1

Determine the transfer function of a system with the following equations.

R, C are positive constants.
i_1, i_2 are time-dependent currents.
$v_i(t)$ is the input and $v_o(t)$ the output p.d.

$$(a) \qquad v_i(t) = 3Ri_1(t) + 2Ri_2(t)$$

$$(b) \qquad Ri_1(t) = Ri_2 + \frac{1}{C}\int_0^t i_2(t)\mathrm{d}t$$

$$(c) \qquad v_o(t) = \frac{1}{C}\int_0^t i_2(t)\mathrm{d}t + R\big(i_1(t) + i_2(t)\big)$$

Example 2

Determine the transfer function of a system with the following equations.

C, R are positive constants.
i_1, i_2, i_3 are time-dependent currents.
$v_i(t)$ is the input and $v_o(t)$ the output p.d.

$$(a) \qquad v_i = R(i_1 + i_2 + i_3) + R(i_2 + i_3)$$

$$(b) \qquad \frac{1}{C}\int_0^t i_1\mathrm{d}t = R(i_2 + i_3)$$

$$(c) \qquad \frac{1}{C}\int_0^t i_2\mathrm{d}t = 2Ri_3$$

$$(d) \qquad v_o = -2Ri_3$$

Response function

The response function $g(t)$ is the inverse Laplace transform of the transfer function $G(s)$.