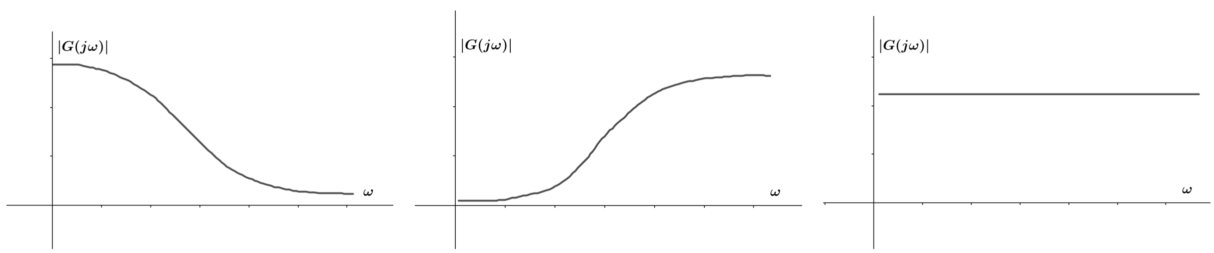
FMSS: Lecture 5 handout

Amplitude Bode Plots

To sketch the amplitude bode plot from the transfer function $G(s)$:

1. Replace s with $j\omega$.
2. Consider the **absolute value** of this function $|G(j\omega)|$
3. How does it behave when ω is extremely **small** ($\omega << 1$)?
4. How does it behave when ω is extremely **large** ($\omega >> 1$)?
5. Draw a rough sketch of $|G(j\omega)|$ against frequency ω.



(a) Low-pass (b) High-pass (c) All-pass

Periodic functions

The minimum time required for one full cycle is the **period**.

The **angular frequency** ω is related to the period T by:

$$\omega = \frac{2\pi}{T} \qquad \text{or} \qquad T = \frac{2\pi}{\omega}$$

Odd and even functions

An **odd** function has rotational symmetry of 180° about the origin, and satisfies $f(-x) = -f(x)$. Sine waves are odd.

An **even** function has reflective symmetry about the vertical axis, and satisfies $f(-x) = f(x)$. Cosine waves are even.

What is Fourier Series?

We can represent (almost) any periodic function by some **combination of sine and cosine waves of different frequencies**.

Given a periodic function $f(t)$ with period T and angular frequency ω, it can be represented by a *unique* series of sines and cosines plus a constant term:

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\omega t) + \sum_{n=1}^{\infty} b_n \sin(n\omega t)$$

where a_0, a_n and b_n are constants, called the **Fourier coefficients**.

The terms in the series have certain names:

- $\frac{1}{2}a_0$ is the **DC level** of $f(t)$.
- $a_1 \cos(\omega t) + b_1 \sin(\omega t)$ is the first harmonic, or **Fundamental mode**.
- $a_n \cos(n\omega t) + b_n \sin(n\omega t)$ is the $\mathbf{n^{th}}$ **Harmonic**. It has angular frequency $n\omega$.

The N^{th} **Partial Sum** for a function $f(t)$ consists of the first N terms of the Fourier Series (count each harmonic as a single term):

$$f_N(t) = \frac{a_0}{2} + \sum_{n=1}^{N-1} \Big(a_n \cos(n\omega t) + b_n \sin(n\omega t) \Big)$$

As we increase the number of terms, the partial sum converges towards the full (infinite) Fourier Series and thus better approximates the true function $f(t)$.

The goal of Fourier analysis is to find the values of the coefficients for a given function $f(t)$.