FMSS: Lecture 6 handout

Complex form of the Fourier Series of $f(t)$

A periodic function $f(t)$ with angular frequency ω can be described by:

$$f(t) = \frac{a_0}{2} + Re\left\{ \sum_{n=1}^{\infty} A_n \, e^{jn\omega t} \right\}$$

This has only **one** sequence of coefficients - the **phasors** A_n

Relationship between phasors and real coefficients

$$A_n = a_n - jb_n, \qquad a_n = Re\{A_n\}, \qquad \text{and} \qquad b_n = -Im\{A_n\}$$

Determining the complex Fourier Series of $f(t)$ with period T

1. Define

$$g(t) = \begin{cases} f(t) & \text{for } 0 < t < T, \\ 0 & \text{otherwise.} \end{cases}$$

2. Obtain the Laplace transform $\bar{g}(s)$.
3. Change the variable to obtain $\bar{g}(jn\omega)$.
4. The phasor of the n^{th} harmonic is then:

$$A_n = \frac{2}{T}\bar{g}(jn\omega)$$

5. Find the DC level by either usual method.
6. State the complex form of the Fourier Series:

$$f(t) = \frac{a_0}{2} + Re\left\{ \sum_{n=1}^{\infty} A_n \, e^{jn\omega t} \right\} \qquad \text{where} \qquad \omega = \frac{2\pi}{T}$$

Special exponential values

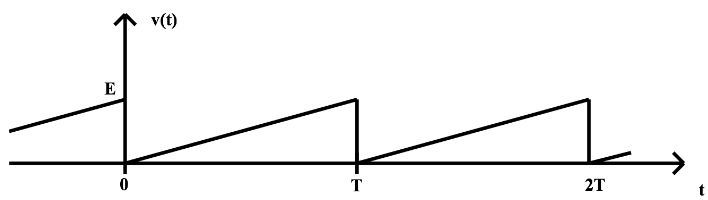
For any integer (whole number) n:

$$e^{2n\pi j} = \cos(2n\pi) + j\sin(2n\pi) = 1 + j \times 0 = 1$$

and

$$\begin{aligned} e^{n\pi j} &= \cos(n\pi) + j\sin(n\pi) \\ &= \cos(n\pi) + j \times 0 = \cos(n\pi) = (-1)^n = \begin{cases} -1 & \text{for odd } n, \\ +1 & \text{for even } n. \end{cases} \end{aligned}$$

Example 1 (Sawtooth Wave)



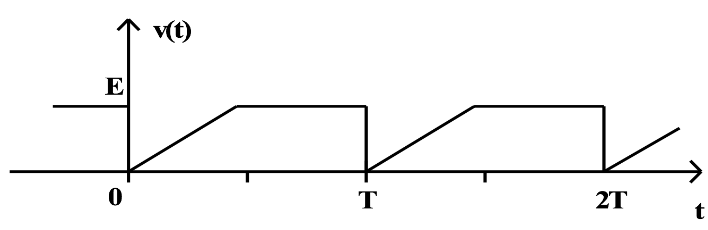
In this example, the wave obeys

$$f(t) = \frac{Et}{T}$$

during the first cycle $0 < t < T$.

Determine the Fourier Series using phasors and Laplace transforms.

Example 2 (Clipped Sawtooth Wave)



In the first cycle $0 < t < T$, this wave $f(t)$ is given by:

$$f(t) = \begin{cases} \frac{2Et}{T} & \text{for } 0 < t < T/2, \\ E & \text{for } T/2 < t < T. \end{cases}$$

Determine the Fourier Series using phasors and Laplace transforms.