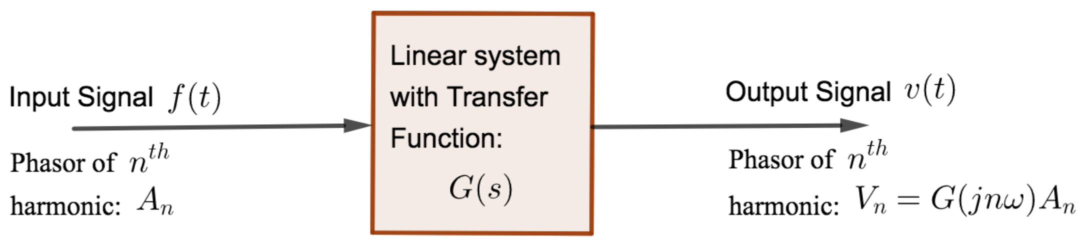
FMSS: Lecture 7 handout

Effect of a linear system on a signal



For a linear system with transfer function $G(s)$ and an input signal $f(t)$ . . .

Frequency response function

For the n^{th} harmonic, this is $G(jn\omega)$.

Output DC level

DC level of output $=$ $G(0)$ $\times$ DC level of input

Phasor V_n of the n^{th} harmonic of the output

To obtain this, the phasor A_n of the input is scaled by the frequency transfer function:

$$V_n = G(jn\omega) \times A_n$$

Output Fourier series

Putting it all together, if the input signal $f(t)$ has complex Fourier series:

$$f(t) = \frac{a_0}{2} + Re\left\{ \sum_{n=1}^{\infty} A_n \, \mathrm{e}^{jn\omega t} \right\}$$

Then the output signal $v(t)$ has complex Fourier series:

$$v(t) = G(0)\frac{a_0}{2} + Re\left\{ \sum_{n=1}^{\infty} G(jn\omega) A_n \, \mathrm{e}^{jn\omega t} \right\}$$

Distortion

Amplitude Distortion: If the modulus of $G(jn\omega)$ depends on n, the amplitude of each harmonic may be scaled by a different factor.

Phase Distortion: If the phase angle of $G(jn\omega)$ depends on n, each phase angle may be altered by a different amount.

Example 1

Consider a system which has transfer function:

$$G(s) = 2\left\{\frac{1+sCR}{1+2sCR}\right\}$$

Determine the complex Fourier series of the output signal $v(t)$, when the input is a pulse wave of period T, which has phasor

$$A_n = \frac{-jE}{\pi n}\big(1-\cos(\pi n)\big) = \begin{cases} \frac{-2Ej}{\pi n} & \text{for odd } n, \\ 0 & \text{for even } n, \end{cases}$$

and DC level $\frac{E}{2}$.

Example 2

Consider a linear system with the corresponding set of equations:

$$e(t) = R(i_1(t)+i_2(t)) + Ri_2(t)$$

$$\frac{1}{C}\int_0^t i_1(t)\mathrm{d}t = Ri_2(t)$$

$$v(t) = Ri_2(t)$$

where R and C are positive constants, $e(t)$ is the input signal, $v(t)$ is the output signal, and $i_1(t)$ and $i_2(t)$ are time-dependent currents.

First, we will determine the transfer function $G(s)$. Then determine the complex Fourier series of the output signal $v(t)$, given a general input:

$$e(t) = \frac{a_o}{2} + Re\left\{\sum_{n=1}^{\infty} A_n\,\mathrm{e}^{jn\omega t}\right\}$$

and then when the input is specifically a sawtooth wave with

$$\text{Phasor:}\quad A_n = \frac{Ej}{\pi n} \qquad\qquad \text{and DC level:}\quad \frac{a_0}{2} = \frac{E}{2}$$