

# Solving and Transposing Equations

Core topics in Mathematics

Lecture 3

# Learning Outcomes

- Solve equations.
- Transpose equations.

# Solving and Transposing Equations

**Transposition** is the process of rearranging an equation into a different form using logically valid steps.

All steps in transposition originate from a single logical principle:

If two initially equal things are changed in an identical manner, then they must still be equal *after* the change.

This means we can make changes to, say, the left-hand side (LHS) of an equation, provided we make precisely the **same change** to the right-hand side (RHS) also.

# Transposing Equations

General principles are to:

- Get rid of fractions by multiplying.
- Get rid of brackets by expanding.
- Gather all terms with the unknown to one side by addition/subtraction.
- Remove everything else to the other side by addition/subtraction.
- Use division to leave the unknown by itself.

But this is a skill mainly acquired by practising many examples.

# Example 1

Solve  $2x - 4 = 10$  for  $x$ .

First, we add  $+4$  to both sides to remove the  $-4$  term on the LHS:

$$2x - 4 + 4 = 10 + 4$$

$$2x = 14 \quad \text{Now } 2x \text{ is alone.}$$

$$\frac{2x}{2} = \frac{14}{2} \quad \text{Remove the factor of 2 by division.}$$

$$x = 7$$

## Example 2

Solve  $3x + 4 = 31$ .

$$3x + 4 - 4 = 31 - 4 \quad \text{Subtract away the } +4 \text{ on the LHS.}$$

$$3x = 27 \quad \text{Now the only } x\text{-term is alone.}$$

$$\frac{3x}{3} = \frac{27}{3}$$

$$x = 9$$

## Example 3

Solve  $5x - 6 = 3x - 8$ .

$$5x - 6 - 3x = 3x - 8 - 3x \quad \text{Gather the } x\text{-terms on LHS.}$$

$$2x - 6 = -8$$

$$2x - 6 + 6 = -8 + 6 \quad \text{Remove the other term.}$$

$$2x = -2$$

$$\frac{2x}{2} = \frac{-2}{2} \quad \text{Divide away the factor of 2.}$$

$$x = -1$$

## Example 4 - Part I

$$\text{Solve } 2(3x - 7) + 4(3x + 2) = 6(5x + 9) + 3.$$

First, always expand all the brackets. Then we will gather the  $x$ -terms and the constants together:

$$6x - 14 + 12x + 8 = 30x + 54 + 3$$

$$18x - 6 = 30x + 57$$

$$18x - 6 - 30x = 30x + 57 - 30x$$

$$-12x - 6 = 57$$



## Example 4 - Part II

Now proceeding as in previous examples:

$$-12x - 6 = 57$$

$$-12x - 6 + 6 = 57 + 6$$

$$-12x = 63$$

$$\frac{-12x}{-12} = \frac{63}{-12}$$

$$\therefore x = -\frac{63}{12} \quad \text{or} \quad -5.25$$

## Example 5

Solve  $\mu = u + at$  for  $a$ .

$$\mu - u = u + at - u$$

$$\mu - u = at$$

$$\frac{\mu - u}{t} = \frac{at}{t}$$

$$\frac{\mu - u}{t} = a$$

Or rather:

$$a = \frac{\mu - u}{t}$$

## Example 6 - Part I/III

Solve  $\frac{2+t}{3} = 2(t-k)$  for  $t$ .

$$3 \times \frac{(2+t)}{3} = 3 \times 2(t-k)$$

$$2+t = 6(t-k)$$

$$2+t = 6t - 6k$$

$$2+t-6t = 6t-6k-6t$$

$$2-5t = -6k$$

## Example 6 - Part II/III

$$\text{Solve } \frac{2+t}{3} = 2(t-k) \text{ for } t.$$

$$2 - 5t = -6k$$

$$2 - 5t - 2 = -6k - 2$$

$$-5t = -6k - 2$$

$$\frac{-5t}{-5} = \frac{-6k - 2}{-5}$$

$$t = \frac{-6k - 2}{-5}$$

## Example 6 - Part III/III

The solution could also be written as:

$$t = \frac{6k + 2}{5}$$

Or:

$$t = \frac{2(3k + 1)}{5}$$

Or:

$$t = \frac{2}{5}(3k + 1)$$

There are often multiple ways to present the solution.

## Example 7 - Part I/IV

Solve  $\frac{1}{f} = \frac{1}{u} + \frac{1}{\mu}$  for  $\mu$ .

$$\frac{1}{f} - \frac{1}{u} = \frac{1}{\mu} + \frac{1}{u} - \frac{1}{u}$$

$$\frac{1}{f} - \frac{1}{u} = \frac{1}{\mu}$$

$$\mu \left( \frac{1}{f} - \frac{1}{u} \right) = \mu \frac{1}{\mu}$$

$$\mu \left( \frac{1}{f} - \frac{1}{u} \right) = 1$$

## Example 7 - Part II/IV

Continuing...

$$\mu \left( \frac{1}{f} - \frac{1}{u} \right) = 1$$

$$\frac{\mu \left( \frac{1}{f} - \frac{1}{u} \right)}{\frac{1}{f} - \frac{1}{u}} = \frac{1}{\frac{1}{f} - \frac{1}{u}}$$

$$\mu = \frac{1}{\frac{1}{f} - \frac{1}{u}}$$

## Example 7 - Part III/IV

Let's see an alternative approach to solving  $\frac{1}{f} = \frac{1}{u} + \frac{1}{\mu}$  for  $\mu$ .

$$\mu \times \frac{1}{f} = \mu \times \frac{1}{u} + \mu \times \frac{1}{\mu}$$

$$\frac{\mu}{f} = \frac{\mu}{u} + 1$$

$$\frac{\mu}{f} - \frac{\mu}{u} = \frac{\mu}{u} + 1 - \frac{\mu}{u}$$

$$\frac{\mu}{f} - \frac{\mu}{u} = 1$$



# Example 7 - Part IV/IV

Continuing...

$$\frac{\mu}{f} - \frac{\mu}{u} = 1$$

$$\mu \left( \frac{1}{f} - \frac{1}{u} \right) = 1$$

$$\frac{\mu \left( \frac{1}{f} - \frac{1}{u} \right)}{\frac{1}{f} - \frac{1}{u}} = \frac{1}{\frac{1}{f} - \frac{1}{u}}$$

$$\mu = \frac{1}{\frac{1}{f} - \frac{1}{u}} = \frac{1}{\frac{u-f}{fu}} = \frac{fu}{u-f}$$

## Example 8

Solve  $A = 2\pi r^2 + 2\pi rh$  for  $h$ .

$$A - 2\pi r^2 = 2\pi r^2 + 2\pi rh - 2\pi r^2$$

$$A - 2\pi r^2 = 2\pi rh$$

$$\frac{A - 2\pi r^2}{2\pi r} = \frac{2\pi rh}{2\pi r}$$

$$\frac{A - 2\pi r^2}{2\pi r} = h, \quad \text{so} \quad h = \frac{A - 2\pi r^2}{2\pi r}$$

## Example 9 - Part I

Solve  $T = 2\pi\sqrt{\frac{L}{g}}$  for  $L$ .

$$\frac{T}{2\pi} = \frac{2\pi}{2\pi}\sqrt{\frac{L}{g}}$$

$$\frac{T}{2\pi} = \sqrt{\frac{L}{g}}$$

$$\left(\frac{T}{2\pi}\right)^2 = \left(\sqrt{\frac{L}{g}}\right)^2$$

$$\left(\frac{T}{2\pi}\right)^2 = \frac{L}{g}$$

## Example 9 - Part II

$$\left(\frac{T}{2\pi}\right)^2 = \frac{L}{g}$$

$$\left(\frac{T}{2\pi}\right)^2 \times g = \frac{L}{g} \times g$$

$$g \left(\frac{T}{2\pi}\right)^2 = L$$

Hence,

$$L = g \left(\frac{T}{2\pi}\right)^2$$