MMaD: Lecture 7 handout

Differentiation

Given a function $y = f(x)$, the "derivative of y with respect to x" can be written as:

$$\frac{\mathrm{d}y}{\mathrm{d}x} \quad \text{or} \quad y'$$

This yields the gradient (slope) of the function, equivalent to its rate of change as x changes.

Standard rules of differentiation

For constants a and n:

$$y = ax^n \implies \frac{\mathrm{d}y}{\mathrm{d}x} = anx^{n-1}$$

$$y = ax \implies \frac{\mathrm{d}y}{\mathrm{d}x} = a$$

$$y = a \implies \frac{\mathrm{d}y}{\mathrm{d}x} = 0$$

$$y = \mathrm{e}^x \implies \frac{\mathrm{d}y}{\mathrm{d}x} = \mathrm{e}^x$$

$$y = \sin(x) \implies \frac{\mathrm{d}y}{\mathrm{d}x} = \cos(x)$$

$$y = \cos(x) \implies \frac{\mathrm{d}y}{\mathrm{d}x} = -\sin(x)$$

$$y = \ln(x) \implies \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{x}$$

Linearity

Given two functions f and g,

$$y = f(x) \pm g(x) \implies \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}f}{\mathrm{d}x} \pm \frac{\mathrm{d}g}{\mathrm{d}x} = f'(x) \pm g'(x)$$

If a is a constant, then:

$$y = af(x) \implies \frac{\mathrm{d}y}{\mathrm{d}x} = a\frac{\mathrm{d}f}{\mathrm{d}x} = af'(x)$$

The Product Rule

Suppose $f(x)$ is the product of two functions (i.e. the result of multiplying them together):

$$f(x) = u(x) \cdot v(x)$$

Then the derivative is given by the Product Rule:

$$\frac{\mathrm{d}f(x)}{\mathrm{d}x} = v(x) \cdot \frac{\mathrm{d}u(x)}{\mathrm{d}x} + u(x) \cdot \frac{\mathrm{d}v(x)}{\mathrm{d}x}$$

The Chain Rule

To differentiate composite functions, we use the Chain Rule.

If $y = g(f(x))$, then we write $u = f(x)$ and so $y = g(u)$. Then...

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{d}y}{\mathrm{d}u} \cdot \frac{\mathrm{d}u}{\mathrm{d}x}$$

Maximising and Minimising Functions

To find the maximum and minimum points of a curve $y = f(x)$:

1. Calculate the first derivative $\frac{\mathrm{d}y}{\mathrm{d}x}$.
2. Solve the equation $\frac{\mathrm{d}y}{\mathrm{d}x} = 0$ for x. This tells us the location of points where the gradient is zero (i.e. the stationary points).
3. Calculate the second derivative $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$.
4. Determine the sign of $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}$ at each stationary point, apply the "Second Derivative Test":

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} > 0 \quad \Longrightarrow \quad \text{local minimum}$$

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} < 0 \quad \Longrightarrow \quad \text{local maximum}$$

$$\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = 0 \quad \Longrightarrow \quad \text{no conclusion}$$

MATLAB

Differentiation uses the `diff` command, which requires two arguments: the function to be differentiated, and then the symbolic variable you are differentiating with respect to.

$$\frac{\mathrm{d}}{\mathrm{d}x}\big(\sin(2x)\big)$$

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syms x
diff( sin(2*x), x )
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