Calculating the Fourier Series of a Sawtooth Wave with period 5

In this script we will plot a sawtooth wave and its approximated Fourier series over the interval 0<t<5.

First, clear any previously-assigned variables.

```
clear
```

1. Defining the sawtooth wave f(t) in the first period

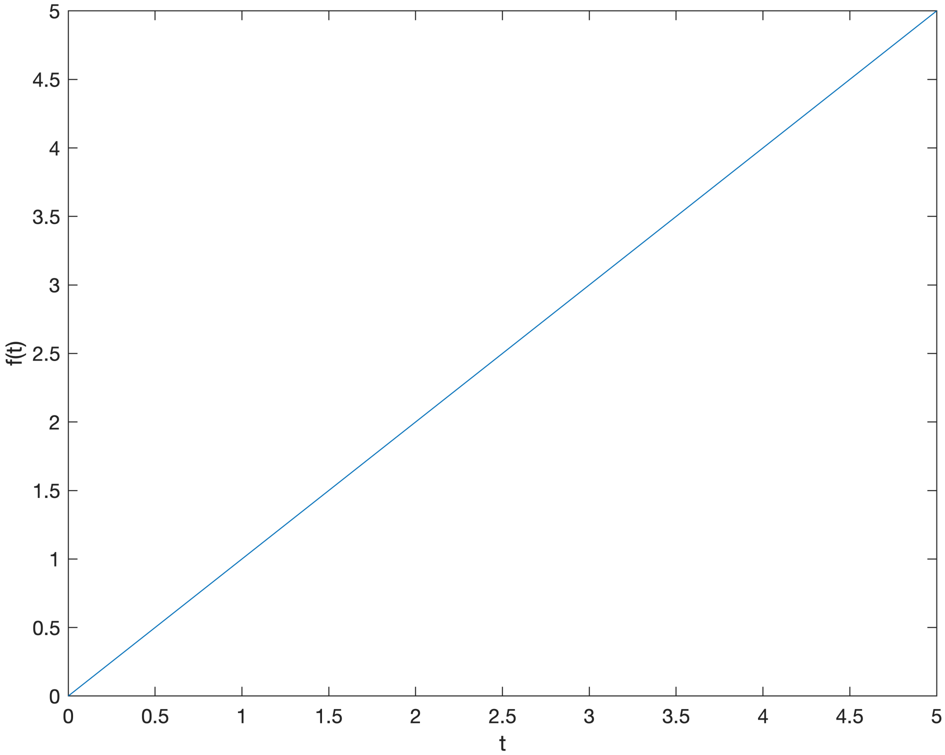
Declare and plot the straight line **f(t)=t** as this is how the function behaves during the first period 0< t < 5.

In this case, we will not need Heaviside step functions, as within one full period there is only one type of behaviour.

```
syms t
f = t
```

f = t

```
fplot(f, [0 5])
xlabel('t')
ylabel('f(t)')
```



2. Calculating the first Fourier coefficients

From the function definition, we are told that this sawtooth wave has period of T = 5.

We need to store that, and calculate the angular frequency.

```
T = 5
```

```
T = 5
```

```
w = 2*pi/T
```

```
w = 1.2566
```

Let's start by calculating the first few **a_k** coefficients of the cosine terms:

```
a0 = int(f, t, 0, T)*2/T
```

```
a0 = 5
```

```
a1 = int(f * cos(w*t), t, 0, T)*2/T
```

a1 = 0

```
a2 = int(f * cos(2 * w * t), t, 0, T)*2/T
```

a2 = 0

```
a3 = int(f * cos(3 * w * t), t, 0, T)*2/T
```

a3 = 0

```
a4 = int(f * cos(4 * w * t), t, 0, T)*2/T
```

a4 = 0

```
a5 = int(f * cos(5 * w * t), t, 0, T)*2/T
```

a5 = 0

See the pattern? All the a_k terms are zero for k>0, so we will not need any cosine terms in the final Fourier series.

Next, let's look at the **b_k** terms:

```
b1 = int(f * sin(t), t, 0, T)*2/T
```

b1 =

$$\frac{2\sin(5)}{5} - 2\cos(5)$$

```
b2 = int(f * sin(2 * w * t), t, 0, T)*2/T
```

b2 =

$$-\frac{5}{2\pi}$$

```
b3 = int(f * sin(3 * w * t), t, 0, T)*2/T
```

b3 =

$$-\frac{5}{3\pi}$$

```
b4 = int(f * sin(4 * w * t), t, 0, T)*2/T
```

b4 =

$$-\frac{5}{4\pi}$$

```
b5 = int(f * sin(5 * w * t), t, 0, T)*2/T
```

```
b5 =
```

$$-\frac{1}{\pi}$$

3. Plotting the approximate Fourier Series

Now we shall construct the first five terms of the Fourier series.

(This is specifically called the **"5th partial sum"**, as the actual Fourier series ought to include all of the infinitely-many terms.)

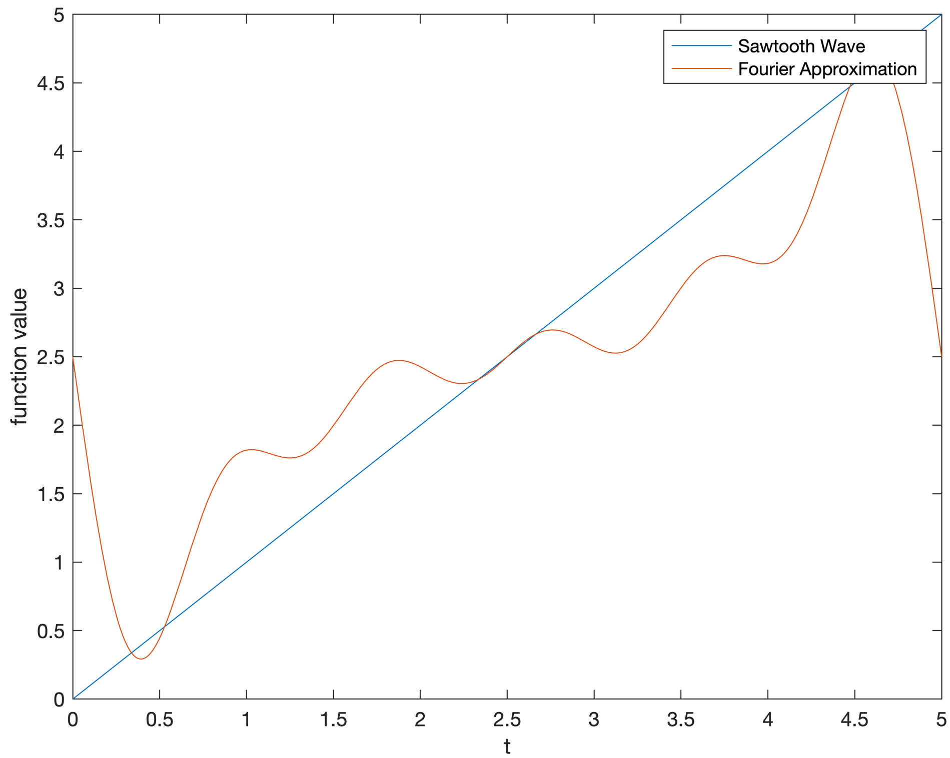
As we know that a_1 , a_2, a_3 ... are all zero we don't need to include these in the sum.

```
FourierApprox = a0/2 + b1 * sin(w*t) + b2 * sin(2 * w * t) + b3 * sin(3 * w * t)...
                + b4 * sin(4 * w * t) + b5 * sin(5 * w * t)
```

```
FourierApprox =
```

$$\frac{5}{2}-\frac{5\sin\left(\frac{4\pi t}{5}\right)}{2\pi}-\frac{5\sin\left(\frac{6\pi t}{5}\right)}{3\pi}-\frac{5\sin\left(\frac{8\pi t}{5}\right)}{4\pi}-\sin\left(\frac{2\pi t}{5}\right)\left(2\cos(5)-\frac{2\sin(5)}{5}\right)-\frac{\sin(2\pi t)}{\pi}$$

```
% Plot both the sawtooth wave and the Fourier partial sum on the same graph.
fplot(f, [0, 5])
hold on
fplot(FourierApprox, [0, 5])
hold off
legend('Sawtooth Wave', 'Fourier Approximation')
xlabel('t')
ylabel('function value')
```



Sawtooth Wave
Fourier Approximation
function value
t
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5