MMaD: Lecture 12 handout

Fourier transform

For a piecewise-continuous periodic function $f(t)$, the inverse Fourier transform may be written:

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega)\, \mathrm{e}^{j\omega t}\, \mathrm{d}\omega$$

and the (forward) Fourier transform is:

$$F(\omega) = \int_{-\infty}^{\infty} f(t)\, \mathrm{e}^{-j\omega t}\, \mathrm{d}t$$

Fourier transform of discrete data

If we have a set of N data points $\{f_n\}$ observed at time $\{t_n\}$, the discrete Fourier transform takes us to the frequency domain:

$$\left(t_n, f_n\right) \xrightarrow{Fourier\ \ transform} \left(\omega_k, F_k\right)$$

Some notation has a different meaning here to other parts of the module:

- t_n are the observation times of our data.
- f_n are the corresponding strengths of the signal observed at time t_n.
- ω_k here refers to frequencies in hertz (*not* angular frequency).
- F_k is the magnitude of corresponding frequency ω_k in the Fourier transform.
- T will refer to the total time interval over which the signal was observed (*not* the period).
- N is the number of samples that were recorded in this time T.

There are N integer values of k, ranging between $-\frac{N}{2}$ and $\frac{N-1}{2}$

For each value of k, there is a specific frequency ω_k, and an associated complex number F_k.

This sequence of numbers F_k, which are independent of the observation time t_n, are the Fourier transform of the sequence $\{f_n\}$.

They are determined by:

$$F_k = \sum_{n=0}^{N-1} f_n\, \mathrm{e}^{-j2\pi nk/N}$$

Using MATLAB or Excel for discrete Fourier transforms

- We will conduct examples of both of these in the tutorial.
- See the MATLAB live script `Lecture11_DiscreteFourierTransformExample.mlx` in the lecture material folder for the workings of the example shown in the lecture slides.
- The main new command for MATLAB is the "fast fourier transform" of the data: `F = fft(f)`

MATLAB procedure

Given a set of N time-dependent data points (t_n, f_n) recorded over a time-period T:

1. Take the Fourier transform of the dependent data (the values of f):

```
F = fft(f)       This takes the Fourier transform
```

2. Take the **magnitudes** of these complex numbers:

```
m = abs(F)       ''abs'' for absolute value
```

3. Calculate the frequency spacing (in Hz) according to:

$$s = \frac{1}{T}$$

4. Plot the frequency spectrum of $(N/2) - 1$ values:

   - On the x-axis, we want $(N/2) - 1$ frequencies that start at 0 and increase by the frequency spacing.
   - On the y-axis, we want the first $(N/2) - 1$ values from the magnitude m.

Nyquist frequency

The maximum frequency we can resolve for a discrete data sample is given by:

$$\frac{N}{2T}$$

where T is the total period of time we have collected data over.