MMaD: Matrices Lecture 1 Handout

Order of a matrix

The order of a matrix is the number of rows $\times$ the number of columns.

$$\begin{pmatrix} 1 & 5 \\ -3 & 2 \\ 2 & 0 \end{pmatrix} \quad \text{is a} \quad 3 \times 2 \quad \text{matrix as it has 3 rows and 2 columns}$$

Only matrices with the **exact same order** can be added or subtracted to each other.

A **square matrix** (order $n \times n$) has the same number of rows and columns.

Matrix multiplication

The number of columns in the first matrix must match the number of rows in the second.

If this is satisfied, the result is is given by the remaining dimensions - the same number of rows as the first matrix and columns as the second matrix.

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \times \begin{pmatrix} 5 \\ 6 \end{pmatrix} = \begin{pmatrix} 1 \times 5 + 2 \times 6 \\ 3 \times 5 + 4 \times 6 \end{pmatrix} = \begin{pmatrix} 17 \\ 39 \end{pmatrix}$$

$$2 \times 2 \qquad 2 \times 1 \qquad\qquad 2 \times 1$$

Identity matrix

The $n \times n$ **identity matrix** has 1's on the diagonal entries and 0's elsewhere.

$$I = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{pmatrix}$$

This is the only matrix which satisfies, for a matrix A of appropriate dimensions,

$$AI = IA = A$$

So it acts like a matrix version of the number "1" when it comes to multiplication.

Transpose and symmetric matrices

The **transpose** of a matrix is obtained by swapping the rows and columns (you can think of this as reflecting across the diagonal axis). It is denoted by a superscript T.

If a square matrix is such that:

$$A^T = A$$

then it is called **symmetric** as it has reflective symmetry about the leading diagonal.

On the other hand, if it is such that:

$$A^T = -A$$

then it is called **antisymmetric**.