MMaD: Matrices Lecture 2 Handout

Determinant

A **square matrix** A has a property called the determinant, denoted by $\det(A)$ or $|A|$.

Determinant of a 2×2 matrix:

$$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$$

Determinant of a 3×3 matrix

Work across the **top row** and multiply each entry by the determinant of the corresponding 2×2 co-matrix of the rows and columns that the current entry is *not* in.

Then change the sign of the middle entry.

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix} = a(ei - fh) - b(di - fg) + c(dh - eg)$$

Properties of the Determinant

Given a square matrix A:

$$\det\left(A^T\right) = \det(A)$$

Given two square matrices A and B of the same size:

$$\det(AB) = \det(A)\det(B)$$

If two rows or columns of A are identical:

$$\det(A) = 0$$

Inverse matrix

For a **square** matrix A, there may exist an inverse matrix A^{-1} such that:

$$AA^{-1} = I \quad \text{and} \quad A^{-1}A = I$$

Inverse of a 2×2 matrix

For a general 2×2 square matrix $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$:

$$A^{-1} = \frac{1}{ad - bc} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix} \qquad \text{or} \qquad A^{-1} = \frac{1}{\det(A)} \begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

If the determinant of a square matrix is **zero**, then that matrix has **no inverse!**

MATLAB commands

To calculate the determinant and inverse of the following matrix in MATLAB:

$$A = \begin{pmatrix} 1 & 2 \\ 3 & -4 \end{pmatrix}$$

We can use the following commands:

`A = [1 2; 3 -4];`	(first declare the matrix A)
`C = inv(A);`	(C is the inverse matrix of A)
`d = det(A);`	(d is the determinant of A)