MMaD: Matrices Lecture 3 Handout

Matrix method for solving two linear simultaneous equations

Given a pair of simultaneous equations:

$$ax + by = p$$

$$cx + dy = q$$

Method:

1. Write the pair of equations as a matrix equation:

$$\begin{pmatrix} ax + by \\ cx + dy \end{pmatrix} = \begin{pmatrix} p \\ q \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} p \\ q \end{pmatrix}$$

$$AX = B$$

2. So the square matrix of coefficients is $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ and the vector X contains x and y which we want to find.
3. Calculate the inverse matrix A^{-1}
4. Pre-multiply both sides by the inverse matrix to obtain X:

$$AX = B \quad \Longrightarrow \quad A^{-1}AX = A^{-1}B \quad \Longrightarrow \quad X = A^{-1}B$$

5. From the entries in vector X, read off the values of x and y.
6. Substitute the values of x and y back into the original equations to verify solutions.

Example 1

Solve for x and y:

$$5x + 2y = 10$$

$$4x - 3y = 14$$

Example 2

Solve for x and y:

$$3x - 5y = 7$$

$$2x + 4y = 20$$