MMaD: Matrices Lecture 4 Handout

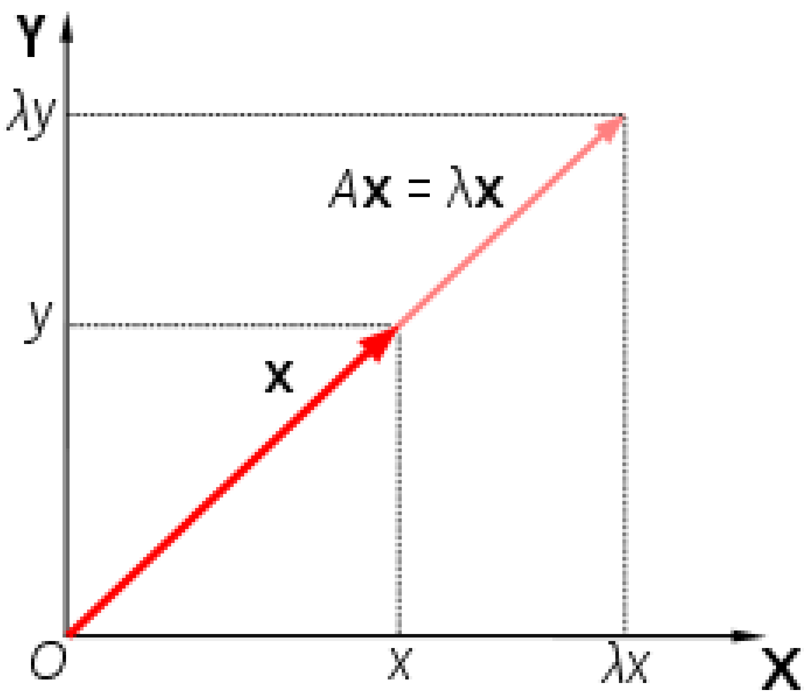
What are eigenvalues and eigenvectors?

When a square matrix A acts on a vector $\underline{\mathbf{x}}$, we obtain a new vector $A\underline{\mathbf{x}}$ that may be stretched and rotated in some way.

It is often useful to find solutions to:

$$A\underline{\mathbf{x}} = \lambda\underline{\mathbf{x}} \quad \text{where} \quad \lambda \quad \text{is a scalar.}$$

These are vectors (**eigenvectors**) $\underline{\mathbf{x}}$ whose direction is **preserved** when we multiply by matrix A. They are magnified by a scaling/magnification factor (**eigenvalue**) λ.



Properties of the eigenvalues and eigenvectors

- For an $n \times n$ square matrix A, there are n eigenvalues (although some may be the same).

- Every eigenvalue has a family of infinitely-many eigenvectors associated with it. They all have the **same direction**, but can be of **any magnitude**.

- This means that if $\underline{\mathbf{e}}_i$ is an eigenvector of A with corresponding eigenvalue λ_i, then so is *any scalar multiple* of $\underline{\mathbf{e}}_i$.

- We will use this to help us find the "easiest" example of an eigenvector in our examples.
- The **sum** of the eigenvalues is equal to the **trace**, which is the sum of the diagonal values.
- The **product** of the eigenvalues is equal to the determinant.

Unit vectors

A **unit vector** has magnitude equal to one.

Given any vector, $\underline{\mathbf{v}}$ we can find the unit vector in the same direction by:

$$\hat{\underline{\mathbf{v}}} = \frac{\underline{\mathbf{v}}}{|\underline{\mathbf{v}}|}$$

In this case, the vertical lines denote the **magnitude** or absolute value of $\underline{\mathbf{v}}$, not the determinant.