MMaD: Matrices Lecture 5 Handout

Calculating eigenvalues and eigenvectors

To obtain the eigenvalue-eigenvector pairs of a square matrix A:

1. First find the eigenvalues by solving the **characteristic polynomial** for λ:

   $$\det\left(A - \lambda I\right) = 0$$

   where I is the identity matrix with the same order as A.

2. Then for each eigenvalue $\lambda = \lambda_1, \lambda_2, \ldots,$ we obtain a corresponding eigenvector $\underline{\mathbf{x}} = \underline{\mathbf{e}}_1, \underline{\mathbf{e}}_2, \ldots$

   We can do this by substituting in the eigenvalue and solving:

   $$A\underline{\mathbf{x}} = \lambda\underline{\mathbf{x}} \qquad \text{for the column vector} \qquad \underline{\mathbf{x}} = \begin{pmatrix} x_1 \\ x_2 \\ \ldots \\ x_n \end{pmatrix}$$

Example: Eigenvalues and Eigenvectors of a 2×2 Matrix

Determine the eigenvalues and eigenvectors of the following 2×2 matrix:

$$A = \begin{pmatrix} 1 & 2 \\ 3 & -4 \end{pmatrix}$$

MATLAB commands

To determine the eigenvalues and eigenvectors of the following square matrix in MATLAB:

$$A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

We can use the following commands:

```
A = [1 2; 3 -4];

B = sym(A);

[vecA,valA] = eig(B);
```

This first creates a symbolic version B and then produces two matrices: $valA$ contains the eigenvalues on the diagonal (with zeros elsewhere) and $vecA$ contains the eigenvectors in each column. The ordering corresponds, so the first column of $vecA$ is the eigenvector corresponding to the eigenvalue in the first diagonal entry of $valA$.