# MMaD: Lecture 1 handout

## Probability of an event

Find the probability of this event $A$ (denoted $P(A)$) by considering the number of possible outcomes where $A$ occurs as a fraction of *all possible outcomes*:

$$P(A) = \frac{\text{No. of outcomes where A occurs}}{\text{No. of possible outcomes}}$$

## "Or" probability

For two events $A$ and $B$, the probability that $A$ **or** $B$ **or** both occur is given by:

> "Or" probability of two generic events:
>
> $$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

If the two events $A$ and $B$ are **mutually exclusive**, they cannot both occur simultaneously. Thus $P(A \text{ and } B) = 0$ and so:

> "Or" probability of two mutually exclusive events:
>
> $$P(A \text{ or } B) = P(A) + P(B)$$

## Conditional events and "and" probability

Two events $A$ and $B$ are **dependent** if the occurrence of $A$ affects the probability of $B$ occuring. In this case, we can talk about the probability of $B$ **given that** $A$ **has already occurred**:

$$P(B|A)$$

Then the probability that *both* $B$ and $A$ occur is:

> "And" probability of dependent events:
>
> $$P(A \text{ and } B) = P(A)P(B|A)$$

If the occurrence of $A$ does *not* affect $B$, they are **independent**:

"And" probability of independent events:

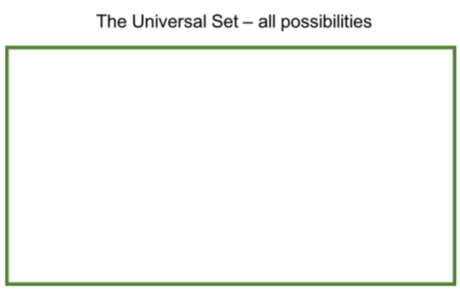
$$P(A \text{ and } B) = P(A)P(B)$$

If the events *are* dependent, we can calculate the conditional probability $P(A|B)$ by thinking of the "pool" of outcomes that we are interested in as being reduced to only those where $B$ has already occured. So for what fraction of *those* events did $A$ occur?

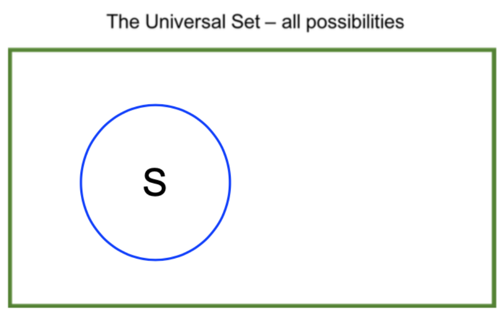
$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}$$

## Venn diagrams

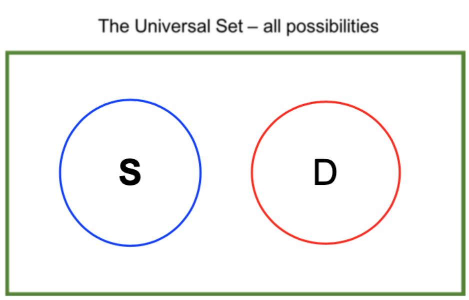
Venn diagrams can help us visualise a subset of outcomes that we are interested in.



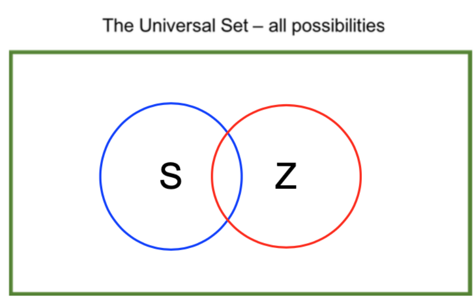

(a) Universal set




(b) $P(S)$




(c) $P(Z \text{ or } D)$




(d) $P(Z \text{ or } S)$

$S$ is the outcome of drawing a spade, $D$ a diamond, and $Z$ a face card. The overlap of the circles in (d) shows that there is a non-zero probability of drawing a card that is *both* a spade and a face card. That is, $P(Z \text{ and } S) \neq 0$.