MMaD: Lecture 6 handout

Discrete probability distributions

A discrete random variable, such as the number X of students in a class, can only take values from some discrete set. A discrete probability distribution would therefore describe the likelihood of X taking various specific discrete values, for example:

$$P(X = 0), \quad P(X = 1), \quad P(X = 2), \quad \ldots$$

Binomial distribution

Consider an experiment which has exactly two possible outcomes:

- "success" with probability p
- "failure" with probability $1 - p$

If we conduct n independent trials, the probability of the number X of successes being exactly equal to $k = 0, 1, \ldots, n$ is given by the **Binomial distribution**:

$$P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$$

where

$$\binom{n}{k} = \frac{n!}{(n - k)!k!}$$

A binomial distribution with n trials and p probability of success has mean number of successes:

$$E(X) = np$$

and variance

$$Var(X) = np(1 - p)$$

Poisson processes

A Poisson process describes a situation where discrete events occur from time to time.

They may be characterised in terms of either:

- the probability of an event happening at each moment in time
- the distribution of the amount of time between two subsequent events.

The specific criteria are:

- Events are independent of each other. The occurrence of one event does not affect the probability another event will occur.
- The **average** rate of events (the number of events per a given time period) is constant.
- Two events cannot occur at the exact same time.

Poisson distribution

For such a process, consider a particular interval of time. If λ is the constant rate at which the event can be expected to occur over that interval, then the probability that the *actual* number of events which occur X in that specific time period is equal to $k = 0, 1, 2, 3, \ldots$ is given by:

$$P(X = k) = \frac{\lambda^k \, \mathrm{e}^{-\lambda}}{k!}$$

Both the mean and the variance of a Poisson distribution are equal to the rate λ.

The distribution of the different values (the probable number of events that occur in that time interval) is given by the Possion distribution.

Example 3

On average, 12 vehicles arrive at an intersection every hour. What is the probability that:

1. Exactly 8 cars arrive in a given hour?
2. Exactly 6 cars arrive in a given 20 minute period?
3. No more than 2 cars arrive in a given hour?